Quality and Quantity, 19 (1985) 145–153
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands

Evolutionary Production Systems

K.H. BRODBECK and R.F. MATZKA
Der Universität München, 8 München 22 (F.R.G.)

Introduction

In modelling economic growth processes it is a common and generally accepted procedure to start from a commodity space of fixed and finite dimension n; different states of the model are then distinguished primarily by the quantities supplied and demanded of the n commodities, and by their prices. The various growth models differ in what is constant and what is variable: population, technology, consumer preferences. However, n, the number of different commodities existing in the economy, remains unexplained and mostly invariant. If invention and innovation are at issue in the literature, as in the tradition of Schumpeter (1952), the introduction of a new commodity is treated as a singular event.

It cannot be denied that in real economies new goods are created constantly, and old goods vanish constantly, and it can hardly be denied that this has important consequences for real economic dynamics. There must be a reason why this feature has been systematically overlooked by present-day growth theory, and we believe this reason to be a methodological one: to question the assumption of a given and fixed commodity space means a challenge to the very grounds of economic reasoning.

Think of an entrepreneur engaged in several productive activities who finds his profits to be below a minimum acceptable level. If he is an inhabitant of a model economy with a fixed commodity space, he may try to raise or lower the levels of some of his activities, or he may engage in one of the n commodities he does not yet produce, in order to improve his profit margin. If no such possibility can be found, he is bound to dissolve his entrepreneurial existence. However, if our entrepreneur inhabits a real economy, it cannot intuitively be deemed "irrational" for him to exhibit some creative imagination, to develop a new good, to produce it, and to sell it to those who find this new good worthwhile buying. It certainly cannot be excluded on a priori grounds that such a strategy may yield acceptable profits.

This example, simple and obvious though it is, transcends two basic

0033-5177/85/$03.30

cornerstones of economic thinking: the concepts of *rationality* and of *equilibrium*. Intuitively, the concepts of rationality and creativity should not be mutually exclusive. However, rationality, as formalized in decision theory and applied in general equilibrium and disequilibrium theory, does explicitly exclude creativity (Matzka, 1982). Rationality, in the formal sense, means the freedom to choose among a *given* set of alternatives in the best possible way, not the freedom to create *new* alternatives. *Equilibrium*, on the other hand, means a state of affairs in which all agents' plans are mutually compatible, so that they can all be realized and all expectations can be fulfilled. Given certain stability conditions, the equilibrium state acts like a centre of gravity: equilibrium tends to be restored endogenously (by the agents' rational reactions) if disturbed exogenously. But what if rational reactions are allowed to include creative reactions? The introduction of a new good could be a stabilizing (or equilibrating) event for an individual agent, but it would certainly mean a disturbance for the system as a whole, to which the rest of the system would have to adjust (cf. the case of new capital goods and a unique rate of profit (Brodbeck, 1983). If creativity were generally accepted as an integral part of rationality, the system would have a tendency constantly to create unforseeable disturbances. The very concept of equilibrium would at once break down, and the concept of disequilibrium together with it (Holub and Matzka, 1982).

We do not have a solution to this dilemma, and do not here enter into a discussion of the methodological issues raised by it. In this paper we attempt to do something much more modest: we circumvent the dilemma by carrying abstraction one step further. We abstract from the quantities as well as the prices of goods, and focus attention on the *number* of *different* goods. That is, we construct a model depicting the dynamics of the dimensionality of the commodity space.

Qualitative Dynamics of an Industrial Production System

COMMODITIES AND PROJECTS

An outstanding characteristic of industrial production systems is that they produce commodities by means of commodities (as highlighted by the title of Sraffa's (1960) famous book). In order to stress this feature, and to simplify the analysis, we here treat land and unskilled labour as given funds available in each period, and do not count them as "commodities". The various kinds of skilled labour and of capital goods are produced by the system and are counted as commodities. This simplification allows the distinction to be abandoned between capital goods and consumption goods

on the one hand, and between capital inputs and labour inputs on the other hand. We shall construct a model on the basis of a discrete time structure, and identify the length of a model period with the period of production. Y_t denotes the set of commodities (more precisely, the set of *species* of commodities) produced in a period t. By the above assumptions, Y_t is also the set of productive factors available in period $t + 1$.

In contrast to bees, who collect honey, or to beavers, who build dams, human beings perform production in their minds before they do so in reality. Humans make plans of what is to be produced, and such a plan may or may not be realized successfully. We therefore introduce the concept of a *project*. By a project we mean a plan to produce a specific kind of commodity, and we denote the set of different projects by X_t. If a produced kind of commodity is formally identified with the plan to produce it, then $Y_t \subseteq X_t$: each commodity is a realized project, but not every project need be realized as a commodity. The quantities y_t and x_t denote the cardinality of the sets Y_t and X_t, respectively, such that $y_t \leqslant x_t$.

INVENTION AND INNOVATION

The number of projects pursued in t must be strongly dependent on the number of commodities produced in $t - 1$. The most simple hypothesis for invention dynamics would therefore be

$$x_t = \alpha y_{t-1}$$

with a rate of invention $\alpha > 1$. This indicates that goods produced in the previous period are pursued as projects subsequently, while creative activities add a certain percentage of newly invented projects. However, not every commodity produced need be pursued as a project subsequently, and creative activities will probably be dampened by the experience of projects not realized in the past. We therefore add a negative damping term, thus generating [1]

$$x_t = \alpha y_{t-1} - \beta(x_{t-1} - y_{t-1}) \tag{1}$$

with a damping rate $\beta > 0$. There are a considerable number of reasons why a project of period t might not be realized, such as lack of technical knowledge (Brodbeck, 1981, pp. 13–20), insufficient expected demand, and the like.

In order to get a simple and closed model on a purely qualitative level of abstraction, we concentrate on one of these reasons: the realization of a project might require certain kinds of productive factors not all of which are actually available. For any project in X_t, then, the likelihood of its being realized is positively related to the number of factors available, that is, to

y_{t-1}. The number y_t of commodities produced will therefore be a fraction of x_t, the magnitude of this fraction being an increasing function of y_{t-1}. This yields, as a innovation mechanism,

$$y_t = x_t k(y_{t-1}) \tag{2}$$

We assume the function $k(y)$ (rate of innovation) to have the following properties: $0 < k(y) < 1$; $k'(y) > 0$; $k''(y) < 0$; $k(0) = 0$; $\lim_{y \to \infty} k(y) = 1$.

MODEL DYNAMICS

Equations (1) and (2) have precisely one nonzero stationary solution, which can be calculated explicitly as

$$x^* = [(\alpha + \beta)/(1 + \beta)]$$
$$y^* = k^{-1}[(1 + \beta)/(\alpha + \beta)] \tag{3}$$

In order to study the system dynamics near this stationary solution, we introduce deviation variables $y_t' = y_t - y^*$, $x_t' = x_t - x^*$, and calculate the linear approximation to eqns. (1) and (2):

$$y_t' = x^* k'^* y_{t-1}' + k^* x_t' \tag{4}$$

$$x_t' = (\alpha + \beta) y_{t-1}' - \beta x_{t-1}' \tag{5}$$

with $k'^* = k'(y^*)$ and $k^* = k(y^*)$. Substituting for x_t, eqns. (4) and (5) can be written in matrix form as

$$\begin{bmatrix} y_t' \\ x_t' \end{bmatrix} = \begin{bmatrix} x^* k'^* + (\alpha + \beta) k^* & -\beta k^* \\ \alpha + \beta & -\beta \end{bmatrix} \begin{bmatrix} y_{t-1}' \\ x_{t-1}' \end{bmatrix} \tag{6}$$

Now observe that $k^* = (1 + \beta)/(\alpha + \beta)$ and calculate the trace Tr and determinant D of the above system matrix:

$$Tr = 1 + x^* k'^*$$
$$D = -\beta x^* k'^* \tag{7}$$

The characteristic equation now reads

$$\lambda^2 - \lambda Tr + D = 0 \tag{8}$$

giving the eigenvalues

$$\lambda_{1,2} = (1/2)\left[Tr \pm (Tr^2 - 4D)^{1/2}\right] \tag{9}$$

and the corresponding eigenvectors

$$e_{1,2} = \begin{bmatrix} \lambda_{1,2} + \beta \\ \alpha + \beta \end{bmatrix} \tag{10}$$

Since $D < 0$, it can be seen at once from eqn. (9) that both eigenvalues are real, and that one of them is positive and the other negative. Furthermore, it may be shown that:

$$\lambda_1 > 1 \tag{11}$$

and that

$$\lambda_2 > -1 \tag{12}$$

if and only if $\beta < 1 + 2/x^* k'^*$.

Proof of eqn. (11). It may at once be verified that inequality (11) is tantamount to

$$(Tr^2 - 4D)^{1/2} > 2 - Tr \tag{13}$$

In the case that $Tr > 2$ this is trivially true. In the opposite case, eqn. (13) may be raised to the power of two. It is then equivalent to $Tr - D > 1$, which is true by eqn. (7).

Proof of eqn. (12). The inequality $\lambda_2 > -1$ is equivalent to

$$(Tr^2 - 4D)^{1/2} < Tr + 2 \tag{14}$$

Since the right-hand side is positive, eqn. (14) may be raised to the power of two. It is then equivalent to $Tr + D > -1$. Substitution for Tr and D from eqn. (7) then yields the condition stated. If it is assumed that $\beta < 1$, which seems not implausible, then this condition is met.

Since the first eigenvalue exceeds unity, the nonzero stationary solution is unstable. The dynamics of the linearized system are visualized in Fig. 1, where we have drawn the eigenvectors (eqn. (10)) and two typical time-paths. The second eigenvector indicates a borderline in the y–x plane, such that any time-path starting from the right of this line drifts farther away from the stationary point in the direction of the first eigenvector, and any time-path starting from the left of this borderline drifts away in the opposite direction. This dominant movement is superimposed on a cyclical movement of two periods' length, which vanishes in the long run (provided that the condition in eqn. (11) holds).

Returning now to the original equations, eqns. (1) and (2), two additional observations can be made. Firstly, it is possible to linearize the system in the neighbourhood of the zero stationary point to obtain the eigenvalues 0 and $-\beta$, which shows that the zero stationary solution is stable in the case that $\beta < 1$. Secondly, for large values of y_t the ratio y_t/x_t approaches unity, as can be seen from eqn. (2). Therefore, the dynamics of the original system are quite easy to understand: a time-path for (x_t, y_t) either approaches the zero stationary state, or tends to a kind of "steady-state" growth with growth rate $\alpha - 1$ and with nearly all projects realized in the long run. Which of these two cases occurs depends on the initial conditions, i.e., on whether the initial point lies to the left or right of some borderline, which must be a nonlinear version of the line B–B in Fig. 1.

We conclude the mathematical analysis of the present model by studying the impact of parameter variations on the stationary solution. Taking differentials in eqn. (3), the following multipliers are obtained:

$$\partial x^*/\partial\alpha = -k^*(1-\eta^*)/(1+\beta)k'^* < 0$$

$$\partial x^*/\partial\beta = (1-k^*)(1-\eta^*)/(1+\beta)k'^* > 0$$

$$\partial y^*/\partial\alpha = -k^*/(\alpha+\beta)k'^* < 0$$

$$\partial y^*/\partial\beta = (1-k^*)/(\alpha+\beta)k'^* > 0$$

where we have written η^* for the elasticity of k at y^* (which is less than unity by our assumptions). An increase in the rate of invention reduces both x^* and y^*, and shifts the borderline to the left; an increase in the damping rate has the opposite effect.

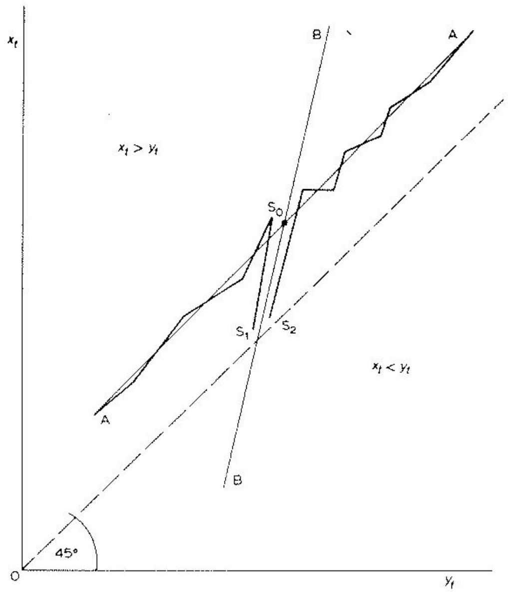

Fig. 1. Dynamics of the Linearized System.

Economic Development—An Evolutionary Process

The model developed in the previous Section is based on rather heroic assumptions, and should therefore be interpreted with great care. For instance, the dynamic equations are based merely on the cardinality of the sets X_t and Y_t, and not on the content of these sets. It would, of course, be nonsense to compare two economic systems by just counting the numbers of different commodities and projects, and to predict similar qualitative futures if these numbers happened to be of similar size. If the equations make sense at all, they do so only in the context of a specific country in its specific historical context [2].

Consider the nonzero stationary solution of the model. If the numbers of commodities and projects happen to obtain precisely their stationary values y^* and x^*, these numbers remain constant over time. It must be emphasized that this solution must not be identified with a state of affairs in which the sets of commodities and projects are kept constant. Since we have defined a project in X_t as a plan actually pursued in period t, such an interpretation would imply that a constant and nonempty set $X \setminus Y$ of idle (i.e., nonrealizable) projects would be pursued permanently, which does not make good sense. Thus, even in the nonzero stationary state, the sets of commodities and projects must change, such that a permanent change in commodity qualities is an unavoidable characteristic of the model economy [3]. If "evolution is the history of a system undergoing irreversible changes" (Lotka, 1956, p. 24; cf. Georgescu-Roegen, 1971), economic growth is essentially an evolutionary process.

The most striking result of the above analysis was a division of the y–x plane into two regions, separated by a borderline, such that the dynamics of a time-path depend crucially on whether the initial point lies to the left or to the right of this borderline. Since the model depicts the dynamics of a purely industrial production system, the question of how an industrial system comes into being is, of course, beyond the scope of the model. We may imagine a purely agrarian economy, which gradually increases the productivity and complexity of its technology such that industrial modes of production evolve, at first using productive inputs produced by the agrarian sector. The dynamic analysis then seems to indicate that the industrial sector can act as a self-maintaining sector only if a certain degree of complexity has been reached (i.e., if the borderline has been crossed).

If economic policy were such as to stress industrial production (at the cost of neglecting the agrarian sector) too early, the consequences would be fatal: the complexity of the industrial system would decline rapidly and the society would be thrown back to the agrarian mode of production. If, on the other hand, the borderline of qualitative complexity has been crossed, the industrial sector is able to maintain itself, and the agrarian sector may decline to minor importance (this seems to confirm Rostow's "take-off" conjecture (Rostow, 1971, Chaps. 3 and 4). The complexity of the industrial system will then essentially increase in geometric progression. In the early days after take-off the system may undergo wild cyclical fluctuations, with periods of rapid innovative activity and periods of technical stagnation, even recession.

Comparing different industrial systems, it is found that take-off is easier for systems with a higher invention rate or a lower damping rate, since, as indicated previously, a rise in α or a fall in β shifts the borderline to the left. This means that for a country with a high innovation rate or a low damping rate, the initial number of different goods which the agrarian system has to provide for take-off to be possible is relatively small.

Concluding remarks

Economic evolution, as well as evolution in general, is clearly a nonmechanical process which, of course, cannot be captured by writing down two simple equations. If nonetheless an attempt is made to do so, this can only be excused as a first tentative step in an unexplored direction. The principal aim was to call attention to the fact that economic growth and development necessarily constitute a process of qualitative change which cannot be described by the methods of conventional equilibrium theory.

Acknowledgement

The authors acknowledge helpful comments by an anonymous referee. The responsibility for the contents of this paper is, of course, the authors' own.

Notes

1 Of course, the cardinality of any finite set is a natural number, while in the equations we admit real numbers as values for x_t and y_t. This is a viable approximation, as long as the numbers involved are large. For small values of x_t and y_t (and even more so for negative values) the model loses its interpretability anyway.

2 We must doubt that the "country that is more developed industrially only shows, to the less developed, the image of its own future" (Marx, 1970, p. 12), if qualitative changes are recognized as creative acts.

3 This "historical" property of our dynamic system is contradictory to Samuelson's definition that "the historical movement of a system may not be dynamical" (Samuelson, 1974, p. 314).

References

Brodbeck, K.H. (1981). *Produktion, Arbeitsteilung und technischer Wandel (Production, Division of Labour and Technical Change)*. Düsseldorf: Volkswirtschaftliche Schriften.

Brodbeck, K.H. (1983). "Neue Kapitalgüter, unvollkommene Konkurrenz und Profitrate (New capital goods, imperfect competition and the rate of profit)", *Zeitschrift für die gesamte Staatswissenschaft* 139: 131–145.

Georgescu-Roegen, N. (1971). *The Entropy Law and the Economic Process*. London: Cambridge University Press.

Holub, H.W. and Matzka, R.F. (1982). "Principles of construction of an economic conflict theory", *Quality and Quantity* 16: 29–42.

Lotka, A.J. (1956). *Elements of Mathematical Biology*. New York: Dover Publications.

Marx, K. (1970). *Das Kapital. Marx – Engels-Werke (MEW), Bd. 23*. Berlin: Dietz Verlag.

Matzka, R.F. (1982). *Struktur und Interpretation der elementaren Nutzen- und Verhandlungstheorie (Structure and Interpretation of Utility Theory and Bargaining Theory)*. Berlin M + M Wissenschaftsverlag.

Rostow, W.W. (1971). *The Stages of Economic Growth*. Cambridge: Cambridge University Press.

Samuelson, P.A. (1974). *Foundations of Economic Analysis*. New York: Atheneum.

Schumpeter, J. (1952). *Theorie der wirtschaftlichen Entwicklung (Theory of Economic Development)*. Berlin: Düncker and Hümblot.

Sraffa, P. (1960). *Production of Commodities by Means of Commodities*. Cambridge: Cambridge University Press.