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Short Articles/*Kurzbeiträge*

Two Class Economies With Overlapping Generations And Heritable Capital Stock

by

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1. Introduction

Two class economies have been discussed in various papers (cf. PASINETTI [1974, 1983]). The "Pasinetti-Paradox" states that in the steady state the rate of profit r is exclusively determined by the equation $s_c r = g$ (s_c = capitalists' saving rate, g = natural rate of growth). The crucial condition for this result is $s_c > s_w$ (s_w = workers' saving rate) (SAMUELSON and MODIGLIANI [1966a]). Otherwise the capital stock owned by capitalists declines whenever it will become positive ($K_c \to 0$) ("Anti-Pasinetti-Case") (SAMUELSON and MODIGLIANI [1966b], PASINETTI [1966], KALDOR [1966], MORISHIMA [1969], p. 34–43); the long term equilibrium in this case can be described in a similar way to that of a Solow-type growth model (SOLOW [1956]). The possibility of pure profit income supposes a right of succession that transfers accumulated capital stock to the next generation of capitalists. For workers there is no such similar condition. To make a consistent analysis of two class economies, some explicit assumptions must be made about the reproduction of the generation of workers as well as capitalists.

In models with overlapping generations, which were developed in the discussion of SAMUELSON's seminal paper [1958], the question of different *social* income classes was not considered. In the approaches of DIAMOND [1966], STEIN [1969], and STARRETT [1972], only the older generation of workers earns profit income. Heterogenities are due to different consumer preferences in a traditional neoclassical context (CASS, OKUNO and ZILCHA [1979]). Further extensions of the overlapping generations approach are due to inheritance of capital (BARRO [1974], BLANCHARD [1985]) and uncertain lifetimes (ABEL [1986]). STEIN [1969] introduces a *social capital* in a neoclassical growth model with overlapping generations that he also interprets as inherited capital. This assumption is

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unusuable for our approach since this capital cannot be consumed (STEIN [1969], p. 144). As Kaldor points out, one cannot assume that shareholders consume only their dividends; every generation uses a part of the capital stock for consumer purposes as well (KALDOR [1956, 1966]).

In the overlapping generations approach, consumption is described in the tradition of the Fisher-Friedman life-cycle-theory. The present paper deals with a situation which is intermediate between the Keynesian interpretation of consumption as a fraction of income and consumption out of wealth (cf. DIAMOND [1966] and BLANCHARD [1985]). Although our analysis seems to be complicated by the fact of different generations and the inheritance of capital, findings for steady states will be reached which are similar to those of Kaldor and Pasinetti.

2. *Assumptions and the Model*

Workers and capitalists live for two periods of equal length. In the first part of their lives, workers are active: in other words, they earn their wages w_t (per worker), save a constant fraction s_L, and spend this capital plus interest in the second passive phase of their lives. To simplify workers are assumed not to inherit capital. [1] An alternative to this interpretation would be that savings are a compulsory contribution to annuity insurance, and are regained from the accumulated contributions plus interest on them. For the savings of the workers we have

(1) $$S_{Lt}^0 = s_L w_t .$$

The upper case 0 and 1 indicate the generation, t is time. With a uniform rate of profit r_t and a constant natural rate of growth g, the stock of capital (per worker) for the next period may be represented by:

(2) $$k_{Lt+1}^1 = S_{Lt}^0 (1+g)^{-1} = s_L w_t (1+g)^{-1} .$$

The savings S_{Lt}^1 of the older generation of workers are zero.

Turn now to the capitalists. While it could be assumed that some of the capitalists also work, this would yield no new insights. In our model world, the capitalists live exclusively from capital income. The new generation inherits a capital stock k_{ct}^0 per worker and savings out of this sum (plus interest)

(3) $$S_{ct}^0 = s_c^0 k_{ct}^0 (1 + r_t) .$$

[1] See the appendix for further explication of this point.

The property rights gained through these savings yield for the older generation a capital stock (per worker)

(4) $$k_{ct+1}^1 = S_t^0 (1+g)^{-1} = s_c^0 k_{ct}^0 (1+r_t)(1+g)^{-1} .$$

The gross income of the older capitalists consists of the accumulated capital plus interest. A part of this gross income is handed down to the next generation. We will describe this formally as the savings of the older generation of capitalists,

(5) $$S_{ct}^1 = s_c^1 k_{ct}^1 (1 + r_t) .$$

These savings become the capital stock for the next generation of capitalists

(6) $$k_{ct+1}^0 = S_{ct}^1 (1+g)^{-1} = s_c^1 k_{ct}^1 (1+r_t)(1+g)^{-1} .$$

Transfers from capitalists to workers and vice versa will be excluded. It may also be observed that the *number* of capitalists in our analysis is completely without significance; it is only the size of the capitalists' share of the pie that matters. [2]

To complete the model we must add the identity of savings and investment:

(7) $$k_{t+1}(1+g) = S_{Lt} + S_{ct}^0 + S_{ct}^1 ,$$

whereby the value of the capital stock consists of the sum of the parts

(8) $$k_t = k_{Lt} + k_{ct}^0 + k_{ct}^1 .$$

In the steady state, the capital per worker $k_t = k_{t+1} = k$ and the rate of profit $r = r_{t+1} = r_t$ remains constant. If $k_t \neq k_{t+1}$, the *value* of capital and the rate of profit changes. (Note that equation (7) only holds ex ante.) With constant capital stock per worker, the difference between ex post and ex ante analysis may be ignored.

3. *Long Term Equilibrium*

Let us for the moment disregard questions of stability, and focus our attention on long term equilibrium. In steady state growth the share of capital stock possessed by each social class remains constant; the time dimension is thus set aside. Our model yields two principal solutions comparable to the Pasinetti/Anti-Pasinetti cases. As mentioned above in the Anti-Pasinetti case, pure capital income disappears, and as a result the capitalists can no longer exist. The whole

[2] Note also that the savings from all types of income are gross savings. The older generation of workers spend the interest as well as their capital. The capitalists consume and save part of their whole capital plus interest. In this framework we deal only with property aspects of the shareholders. We do not consider the real depreciation of capital. It is important to bear this in mind for the discussion below.

capital stock is identical with k_L, in other words, $k_L = k$. Our model then degenerates to a one class model similar to that of Diamond, Stein, and Starrett. [3] In the present context, this case is not important.

If, however, the class of capitalists does not disappear, it follows from equation (4) and (6) that

$$k_c^1 = s_c^0 k_c^0 \left(\frac{1+r}{1+g}\right) = s_c^0 s_c^1 \left(\frac{1+r}{1+g}\right)^2 k_c^1 . \tag{9}$$

From this we discover

$$s_c(1+r) = (1+g); \quad s_c = (s_c^0 s_c^1)^{1/2} . \tag{10}$$

The second solution for equation (9) $(1+r)s_c = -(1+g)$ is irrelevant because g cannot be smaller than -100%. The steady state equilibrium therefore is unique. Equation (10) is similar to but not identical with the Kaldor-Pasinetti equation $s_c r = g$.

Disregarding the less interesting cases, in which the class of capitalists completely disappears, the present model supports Pasinetti's thesis that the long-term rate of profit is determined *exclusively* by the savings preferences (of both generations) of capitalists. There is a substantial difference, however, for *negative* growth rates. In the Kaldor-Pasinetti model the propensity to save is defined in relation to gross income, but not disaggregated into the income of different generations. If the populations decreased the rate of profit would be negative. In contrast to Kaldor-Pasinetti, from equation (10) we get

$$r > 0 \quad \text{for: } s_c < 1 - |g| \quad \text{and} \quad g < 0 . \tag{11}$$

A positive rate of profit may be compatible with a negative rate of growth if the geometric mean of both generations of capitalist's propensity to save s_c is sufficiently small.

The "Keynesian" definition of the propensity of both classes to save can be deduced from the savings preferences of each generation. If we define

$$\hat{s}_L = \frac{s_L w}{w + (1+r)k_L}; \quad \hat{s}_c = \frac{s_c^0 k_c^0 + s_c^1 k_c^1}{k_c^0 + k_c^1} \tag{12}$$

[3] The conditions for this case will be given at the end of this section and in the appendix. If there is no capitalist class the remaining workers in our model can be analyzed similar to the approach of identical agents. Assume for the moment a utility function $u(c_1, c_2) = \beta \log c_1 + (1-\beta)\log c_2$ maximized subject to the budget constraint $w - c_1(1+r) \geqq c_2$ for a two period lifetime. The optimal condition then is

$$S_1 = (1-\beta)w .$$

Therefore our s_L is equal to $(1-\beta)$ in the Diamond model with a representative consumer and a preference function of the log-normal typ used in SAMUELSON's paper [1958]. The exact time path of k_c in our model in this case is, however, rather complicated and a topic for another paper.

for average values, for long-term equilibrium it follows from equation (2), (4) and (10) that

$$\hat{s}_L = s_L \frac{1}{1 + s_c s_L}; \quad \hat{s}_c = s_c^0 \frac{s_c + s_c^1}{s_c + s_c^0} . \tag{13}$$

The average propensity of workers to save is therefore dependent on the propensity of capitalists to save. On the contrary, the average propensity of capitalists to save is merely a modification of their different preferences. This shows that an analysis which uses average values does not completely reflect the disaggregated relations.

To complete our analysis we must investigate the stability conditions of the long-term equilibrium. As equation (10) suggests, the propensity of workers to save does not enter into the picture at all. But we must be certain that we are not looking at an Anti-Pasinetti case. We can establish this in two ways. First, we must ensure that workers' savings do not exceed total investment in the long term equilibrium. If this were the case, k_{Lt} would increase and finally match k_t. Secondly we must observe

$$k > k_L . \tag{14}$$

If v is the capital coefficient, obviously

$$w + rk = y = k/v; \quad \text{or} \quad w = k[(1/v) - r] . \tag{15}$$

Because of (2) in connection with equation (14) we find

$$k > s_L w(1+g)^{-1}; \quad \text{or} \quad k > s_L k[(1/v) - r](1+g)^{-1} . \tag{16}$$

From this, the following condition for the stability of long-term equilibrium may be stated:

$$v > \left[(1+g)\left(\frac{1}{s_L} + \frac{1}{s_c}\right) - 1\right]^{-1} \quad \text{and} \quad g > -0.5 . \tag{17}$$ [4]

For empirically relevant values, equation (17) would have to be fulfilled in most cases. Even with a slowly decreasing population, the denominator on the right-hand side of (17) will not be smaller than one. Take e.g. the following figures: $s_L = 0.2$, $s_c = 0.8$, $g = -0.1$. This leads to an equilibrium rate of profit $r = 12.5\%$. The right-hand side of inequality (17) then becomes 0.22. Therefore a capital coefficient greater than one is sufficient for the stability of two class economies with overlapping generations and heritable capital stock.

[4] The propensity to save s is smaller than one. Therefore $\alpha = (1/s_L) + (1/s_c) \geqq 2$. The denominator on the right-hand side of (17) must be positive: $(1+g)\alpha - 1 > 0$. If g is negative this inequality holds for $g > -0.5$, a constraint which does not seem to be very restrictive in the outside world.

4. Conclusions

If one incorporates the concept of inheritability of capital into a model with overlapping generations and two social income classes, various properties of the Kaldor-Pasinetti model may be reproduced. The long-term interest rate is determined exclusively by the savings and heritability preferences of the capitalists as well as the natural growth rate. The profit rate is in general *larger* than the growth rate; that is to say, it violates the golden rule. The difference between our model and the Kaldor-Pasinetti model is that in our model the system provides a solution for a positive rate of interest even with a *declining* population.

If the propensity of workers to save is interpreted as a contribution to their own retirement benefits, this will clearly have no influence on the interest rate – as long as we exclude the Anti-Pasinetti case, in which the capitalists' share holdings are zero. A change in the *law of inheritance*, however, will influence the interest rate. For example, if there is a redistribution from capitalists to workers through a tax on the capital handed down to the next generation of capitalists, the long term interest rate would rise.

A number of differences may be seen when we compare our analysis to the model with overlapping generations. SAMUELSON [1958] and later authors describe the economic process by means of a "representative individual". Through this assumption the *functional* difference between pure capital income and wages is obscured. Samuelson must therefore assume exogenously the income profile of this representative individual.[5] With these premises the equilibrium rate of interest is not unique; *multiple* market equilibria are possible (CASS, OKUNA and ZILCHA [1979], BRODBECK [1985]). If the different income categories are split up, however, as in our analysis, multiple equilibria can no longer exist.

DIAMOND [1965] considers two types of income explicitly, but he transforms the *social* difference between capital and wages into a *temporal* one. The representative individual is a worker during the first period of his/her life, a capitalist during the second. In our approach this is only a special ("Anti-Pasinetti"-)case. This dishwasher-millionaire model implicitly supposes that capital stock is identical to the assets held by the younger generation for retirement. Diamond et al. determine the interest rate neoclassically: Samuelson's "biological" paradox thereby disappears. The reason for this is the division of exogeneous *income profile* into factor incomes. At least for steady states, the two class theory of Kaldor and Pasinetti offers a simpler explanation which also holds with overlapping generations and heritable capital. The two class approach demands further research in situations other than long-term equilibrium.

[5] The income level plays no role in the solution of the equilibrium values. Samuelson only needs to make the assumption that the income profile is independent of the income distribution.

Summary

In analyzing two class economies the present paper assumes heritable capital and overlapping generations of workers and capitalists. If a class of capitalists exists which earns capital income exclusively, their propensity to save determines the rate of profit, given the natural rate of growth. In contrast to the similar findings of Kaldor and Pasinetti, the rate of profit may be positive although the natural growth rate is negative. With empirically plausible assumptions the long-term equilibrium is stable.

Zusammenfassung

Zur Analyse von Zwei-Klassen-Ökonomien wird im vorliegenden Aufsatz ein vererbbarer Kapitalstock mit überlappenden Generationen vorausgesetzt. Wenn man unterstellt, daß eine Klasse von Rentiers ausschließlich Kapitaleinkommen bezieht, so ist der langfristige Zinssatz bei gegebener Wachstumsrate ausschließlich durch die Sparneigungen der beiden Rentiersgenerationen bestimmt. Im Unterschied zu den analogen Ergebnissen Kaldors und Pasinettis kann der Zinssatz auch bei einer sinkenden Bevölkerung positiv sein. Die Stabilität des langfristigen Gleichgewichts ist bei emprisch plausiblen Annahmen garantiert.

Appendix

In our discussion we have assumed that workers do not save during the second part of their lives. Let us now assume the opposite. Using a similar notation to that which we used for capitalists, workers' savings in the long-term equilibrium now read

(A.1) $$s_L^0 = s_L^0[w + (1 + r)k_L^0]$$

(A.2) $$s_L^1 = s_L^1(1 + r)k_L^1 .$$

All values are per worker. If g is the natural rate of growth, the values for the workers' shares of capital in the first and second generation are

(A.3) $$k_L^1 = s_L^0[w + (1 + r)k_L^0](1 + g)^{-1}$$

(A.4) $$k_L^0 = s_L^1(1 + r)k_L^1(1 + g)^{-1} .$$

The inherited capital stock of workers in the long-term equilibrium becomes

(A.5) $$k_L^0 = w s_L^0 s_L^1(1 + r)(1 + g)^{-2}(1 - \beta)^{-1}; \quad \beta = (s_L^0 s_L^1)/(s_c^0 s_c^1) .$$

A necessary condition for the existence of this equilibrium is:

$$k_L^0 > 0 \quad \text{for:} \quad s_c^0 s_c^1 > s_L^0 s_L^1 \text{ or } \beta < 1 . \tag{A.6}$$

In addition to the above analysis, it can be seen that a further condition is needed: the geometric mean of the workers' propensity to save must be smaller than that of the capitalists s_c. This condition is obviously a modification of $s_c > s_w$, which is necessary to exclude the Anti-Pasinetti case in the Pasinetti model (PASINETTI [1962], SAMUELSON and MODIGLIANI [1966]).

Similar to equation (17), the condition for stability demands that savings from wage income must not exceed long-term investment. With (10) equation (A.5) becomes

$$k_L^0 = s_c w\beta(1-\beta)^{-1}(1+g)^{-1} . \tag{A.7}$$

With equations (A.3), (A.4) and (10) we get

$$k_L = k_L^0 + k_L^1 = w(\beta s_c + s_L^0)(1-\beta)^{-1}(1+g)^{-1} . \tag{A.8}$$

We know from equation (14) that $k > k_L$. This means

$$k > w(\beta s_c + s_L^0)(1-\beta)^{-1}(1+g)^{-1} . \tag{A.9}$$

With (15) we find

$$k > [(1/v)\, k(\beta s_c + s_L^0)(1-\beta)^{-1}(1+g)^{-1} , \tag{A.10}$$

or for v

$$v > \left[(1+g)\left(\frac{1}{s_c} + \frac{1-\beta}{s_L^0 + \beta s_c}\right) - 1\right]^{-1} \text{and } |g| > 1 - s_c \text{ for } \beta \leqq 1 . \tag{A.11}$$

(A.11) together with equation (A.6) is the stability condition for a model with heritable capital for both classes. As can easily be shown, with $\beta = 0$ this equation (A.11) is identical to (17). Because the right-hand side of (A.11) exceeds that of inequality (17) for $\beta > 0$ (the case of inheritance within the working class) the stability condition is stronger in this case.

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